1. Give multiple series for \( f(z) = \frac{1}{1+z^2} \) expanded in powers of \( z \) (i.e. \( z_0 = 0 \)) so that each series’ region of validity is disjoint from the others’ region(s) of validity.

2. Give a series for the same \( f(z) = \frac{1}{1+z^2} \) expanded in powers of \( z - i \) (i.e. \( z_0 = i \)), whose region of validity is \( 0 < |z - i| < R \) for some \( R \) (tell what \( R \) is).

3. Find \( \int_C \frac{1}{z-z_0} \, dz \) if \( C \) is the circle \( |z-z_0| = 1 \), oriented positively, using the Cauchy Integral Formula.

4. Find \( \int_C \frac{e^z}{1+z^2} \, dz \) if \( C \) is the circle \( |z - i| = 1 \), oriented positively, using the (extended or not) Cauchy Integral Formula. Explain how the various hypotheses of the Formula are met.

5. Give terms of the power series for \( h(z) = e^{z^2} \sin z \) expanded in powers of \( z \), up to the \( z^7 \) term. Find the easy way to do this.

(a) In your work, explain why the series will only have nonzero coefficients for odd powers of \( z \), even for terms beyond the \( z^7 \) term.

(Addition problems added 4/11 - installment 2)

6. Use your answer in (5) to compute \( h^{(5)}(0) \), that is, the fifth derivative of \( h \) evaluated at 0.

7. Give the Taylor series for \( f(z) = z^4 + 3z^3 - 2z^2 + 4z - 6 \), expanded about \( z_0 = 1 \).

(a) Find and describe an “easy” way to check your work that a (good) precalculus student could carry out. You do not need to actually show the check, unless you want to.

8. Find the residue of \( f(z) = \frac{\sin(z^2)}{z^7} \) at \( z_0 = 0 \).

9. Give the series expansion of \( g(z) = \frac{e^z - 1}{\sin(z) - z} \) “up to” the \( z \) term (include all terms with negative powers of \( z \), the constant term, and the \( z \) term; some of these may be zero.)

(a) Use your answer to compute \( \int_C g(z) \, dz \) where \( z \) is the unit circle, positively oriented.

(b) We will learn an even shorter way to find this integral next week, you can come back and check your answer, but in part (a), show how you used the answer in (9).