

Name \_\_\_\_\_ Score \_\_\_\_\_

Instructions: This exam is closed-book and closed-notes. Anything may be stored electronically on your calculator. Consultation is allowed ONLY with the instructor. Explain and/or show work for credit.

1. Re-write the equation without any logarithms:  $\log(y) = 1.5 \log(x) + 1.2$
2. Simplify and write without any negative or fractional exponents:

(a)  $\left(\frac{3x^2y^{-2}\sqrt{z}}{2x^6y^{-1}z^3}\right)^{1/2}$

(b)  $(x + 2\sqrt{x})(3 - 3\sqrt{x})$

3. Factor each expression as fully as possible:

(a)  $x^4 - x^2$

(b)  $x^2 + 2x - 15$

4. Combine into as few logarithms as possible:

(a)  $\log_5(x) + 2 \log_5(y)$

(b)  $\frac{\log_a(d) - \log_a(b)}{\log_a(c)}$

5. Solve the equations exactly:

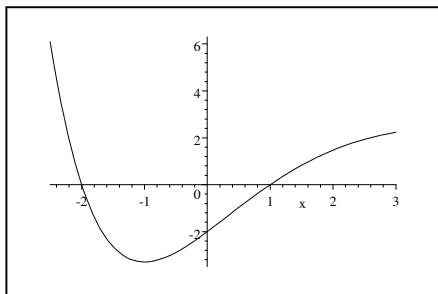
(a)  $3(x - 2) = 5x + 7$

(b)  $4x^2 + 2x - 1 = 0$

6. Find the vertex of the parabola  $y = x^2 - 3x + 5$ ; explain your method.
7. Find approximate solutions to  $x^3 - 2x - 7 = 0$ , and tell how you know you have all the solutions.
8. Find an approximate solution to  $2^x = 4 - x$ .
9. A radioactive isotope decays exponentially from 10 grams to 4 grams in 24 hours. When will only 0.01 grams remain?
10. Give the equation of the line that passes through (3, 8) and is parallel to  $3x + 2y = 4$ .
11. Give an equation for an exponential function whose graph passes through (1, 4) and (3, 9).
12. For each, draw a unit circle and indicate on the circle how it gives the value of:
  - (a)  $\sin(\pi/2)$
  - (b)  $\cos(\pi)$
  - (c)  $\cos(0)$

13. The functions  $f, g$  and  $h$  are as given:

(a)



$y = f(x)$

$x$	$g(x)$
-3	-3
-2	-2.5
-1	-1.5
0	0.5
1	2
2	3
3	2

$$h(x) = \sqrt{5-x}$$

(b) Find each if possible, or say why it is not possible; using the graph may require some visual estimates.

1.  $f(g(0))$       2)  $g(f(0))$       3)  $h(f(2))$       4)  $g(h(-11))$       5)  $\frac{f(1.5) - f(1)}{1.5 - 1}$

(c) Suppose that the entire graph of  $f$  is given above. Estimate the domain and range of  $f$ .

(d) Give the domain of  $h$ .

14. Suppose that  $h(x) = \sqrt{\cos(5-x)}$ , find functions  $f$  and  $g$  so that  $h(x) = f(g(x))$ . There are several acceptable answers, but DO NOT choose  $f(x) = x$  or  $g(x) = x$ .

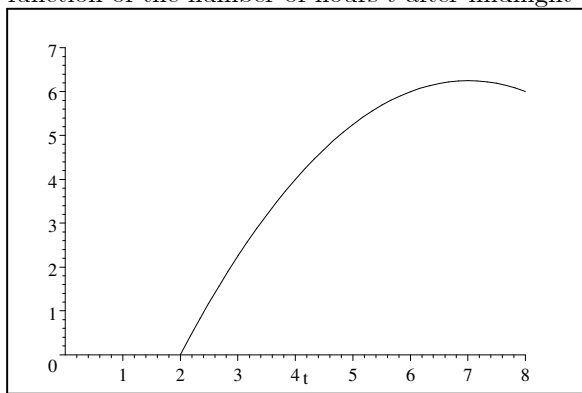
15. The price  $P$  of a bag of fertilizer varies with the number of bags  $N$  that are purchased so that  $P = f(N)$ .

(a) Suppose that  $f^{-1}(6) = 13$ . Tell what this means to someone purchasing fertilizer.

(b) Suppose instead that  $f(N) = 5 + \frac{5}{N}$  dollars. (This contradicts the numbers in (a); don't worry about that). Find a formula for  $f^{-1}$  and give appropriate units.

16. Consider the function  $f(x) = 3^x$ . Give the formula for a function  $g(x)$  so that  $g$ 's graph is found by shifting the graph of  $f$  left 2 units, reflecting about the  $x$  axis, and then shifting up one unit.

17. A snowstorm deposits snow on a town which later melts. The depth  $D$  of the snow, in feet, is given as a function of the number of hours  $t$  after midnight in the following graph.



(a) What is the average rate of change of snow-depth between 2 a.m. and 6 a.m.?

(b) What does your answer in (a) mean in non-mathematical terms?