Directions: Work neatly on a separate sheet of paper. Your group will hand in one write-up with everyone’s name on it. DO NOT fold the corner over to hold everything together! Your final write-up should be very neat and well-written. Remember to use complete sentences as appropriate.

Work together on each problem; do not delegate different problems to different people.

1. Find the dimensions of a rectangle with area 400m$^2$ whose perimeter is as small as possible.

2. Find the point on the line $y = 4x + 7$ that is closest to the origin.

3. An oil refinery is located on the north bank of a straight river that is 2 km wide. A pipeline is to be constructed from the refinery to storage tanks located on the south bank of the river 6 km east of the refinery. The cost of laying pipe is $400,000/km over land and $800,000/km under the river to the tanks. To minimize the cost of the pipeline, how far east should the builders lay pipe over land before going under water to the tanks? [Hint: the distance should be between 0 and 6 km.]

4. A right circular cylinder is inscribed in a sphere of radius $r$. Find the largest possible surface area of such a cylinder.

5. At which points on the curve $y = 1 + 40x^3 - 3x^5$ does the tangent line have the largest slope?

6. Two vertical poles $PQ$ and $ST$ are secured by a rope $PRS$ going from the top of the first pole to a point $R$ on the ground between the poles and then to the top of the second pole as in the figure. Show that the shortest length of such a rope occurs when $\theta_1 = \theta_2$. 