In-Class Assignment 10: Parametric Curves

**Directions:** Work neatly on a separate sheet of paper. Your **group** will hand in one write-up with everyone’s name on it. **DO NOT** fold the corner over to hold everything together!

Work together on each problem; do not delegate different problems to different people.

In mathematics, the word **parameter** has a couple of different meanings. For this worksheet, a parameter is a variable on which the coordinate variables \(x\) and \(y\) depend.

1. Suppose that \(x = 2t\) and \(y = t^2 + 1\).
   
   (a) Make a table of values for \(t, x,\) and \(y\) that includes \(t = -3, -2, -1, 0, 1, 2, 3\).
   
   (b) Plot the points \((x, y)\) from your table.
   
   (c) What kind of curve do you think this represents?
   
   (d) Solve one of the equations for \(t\) and substitute into the other equation to find a relationship between \(x\) and \(y\). Was your hypothesis right in part (c)?

2. Repeat Exercise 1 with \(x = 1 - 2t, y = 3t + 4\).

3. Repeat Exercise 1 with \(x = \cos t, y = \sin t\) (with appropriate choices of \(t\)).

4. Suppose that we mark a point on circle \(O\) having radius \(r\) and then roll the circle, keeping track of the curve the point traces out as the circle rolls. In this problem, we will determine parametric equations for this curve.

   (a) Start with the circle resting on the origin \((0, 0)\), and let the special point \(P\) that we follow be the point of the circle at the origin. We will roll the circle down the positive \(x\)-axis, so sketch the circle in its starting position and after it has rolled for a while, including \(P\) both times. Also label the point \(Q\) at the bottom of the circle (so \(P = Q\) in the starting position).

   (b) The parameter we will use is the angle \(\theta\) through which the circle has rolled: \(\theta = m\angle POQ\).

   (c) An arc of a circle of radius \(r\) subtended by an angle \(\theta\) is \(r\theta\). Use this information to determine the \(x\)-coordinate of \(P\) in terms of \(\theta\).

   (d) Also determine the \(y\)-coordinate of \(P\) in terms of \(\theta\).

   (e) Sketch some points of this curve. (We will ask Maple to sketch the whole curve for us.)