Solutions to Homework Assignment 3

MATH 249
Section Stewart 6e 12.3, Page 784
1-20, 23, 24, 26, 27, 35-40, 45, 47, 53, 58

8. \((4\hat{j} - 3\hat{k}) \cdot (2\hat{i} + 4\hat{j} + 6\hat{k}) = 0(2) + 4(4) + (-3)(6) = -2.\)

14. The dot product gives the vendor’s total income that day from hamburgers, hot dogs, and soft drinks.

16. \(\langle \sqrt{3}, 1 \rangle \cdot \langle 0, 5 \rangle = \sqrt{3}(0) + 1(5) = 5.\) \(|a| = \sqrt{(\sqrt{3})^2 + 1^2} = 2\) and \(|b| = \sqrt{0^2 + 5^2} = 5.\) Thus \(a \cdot b = 5 = (2)(5) \cos \theta,\) so \(\cos \theta = \frac{1}{2}.\) Therefore, \(\theta = 60\) degrees.

24. (a) Since \(u = -\frac{3}{4}v,\) \(u\) and \(v\) are parallel.

(b) \(u \cdot v = 1(2) + (-1)(-1) + (2)(1) = 5.\) This \(u\) and \(v\) are neither parallel nor perpendicular.

(c) \(u \cdot v = a(-b) + b(a) + c(0) = 0,\) so \(u\) and \(v\) are perpendicular.

38. The scalar projection of \(b\) onto \(a\) is \(\frac{\langle -2, 3, -6 \rangle \cdot \langle 5, -1, 4 \rangle}{\langle -2, 3, -6 \rangle} = \frac{-37}{7}.\) The vector projection of \(b\) onto \(a\) is therefore \(\frac{-37}{7} \cdot \frac{-2, 3, -6}{7} = \frac{-37}{49} \cdot \langle -2, 3, -6 \rangle.\)

58. (a) The length of any side of any triangle is less than or equal to the sum of the lengths of the other two sides.

(b)
\[
|a + b|^2 = (a + b) \cdot (a + b)
= a \cdot a + a \cdot b + b \cdot a + b \cdot b
= |a|^2 + 2|a||b| \cos \theta + |b|^2
\leq |a|^2 + 2|a||b| + |b|^2
= (|a| + |b|)^2.
\]

Therefore, \(|a + b| \leq |a| + |b|\).