Solutions to Homework Assignment 4

MATH 249-01 and -02

Section 12.4, Page 820

1-7, 9-12, 14, 15, 25, 26, 30, 31, 34, 35, 36, 45

2. \(< 1, -1, 1 > \times < 2, 4, 6 > = < 1(6) - (-1)(4), -[1(6) - (-1)(2)], 1(4) - (1)(2) > = < 10, -8, 2 >\). You can check the orthogonality.

6. \(< 1, e^t, -e^{-t} > \times < 2, e^t, -e^{-t} > = < e^t(-e^{-t}) - e^{-t}(e^t), -[1(-e^{-t}) - e^{-t}(2)], 1(e^t) - e^t(2) > = < -2, 3e^{-t}, -e^t >\).

13. Only (a), (c), and (f) are meaningful. The others are all attempting to take a cross product in which at least one factor is a scalar.

14. \(|u \times v| = 5 \cdot 10 \sin(60^\circ) = 25\sqrt{3}\). According to the right-hand rule, it will be directed into the page.

16. (a) Since the \(z\)-axis is perpendicular to the \(xy\)-plane, we have \(|a \times b| = 3(2) \sin(90) = 6\).

(b) The cross product is in the \(xy\)-plane, so the \(z\)-component is 0. It is also going to be in the second quadrant (using the right-hand rule), so its \(x\)-component is positive and its \(y\)-component is negative.

18. \(< 3, 1, 2 > \times (< -1, 1, 0 > \times < 0, 0, -4 >) = < 3, 1, 2 > \times < -4, 0, -4 > = < 8, -8, -8 >\), while \(< 3, 1, 2 > \times < -1, 1, 0 > \times < 0, 0, -4 > = < -2, 2, 4 > \times < 0, 0, -4 > = < 8, -8, 0 >\).

30. \(\vec{PQ} = < -3, 2, -1 >\) and \(\vec{PR} = < 1, -1, 1 >\). A vector orthogonal to both of these is also orthogonal to the plane through \(P, Q,\) and \(R\). Such a vector is \(< -3, 2, -1 > \times < 1, -1, 1 > = < 1, 2, 1 >\). The area of the parallelogram the three points defined is the norm of this, or \(\sqrt{6}\), so the area of the triangle is half that: \(\sqrt{6}/2\).

35. We need vectors representing the edges: \(u = \vec{PQ} = < 2, 1, 1 >\), \(v = \vec{PR} = < 1, -1, 2 >\), and \(w = \vec{PS} = < 0, -2, 3 >\). The volume of the parallelepiped is \(|u \times (v \times w)| = | < 2, 1, 1 > \times < 1, -1, 2 > \times < 0, -2, 3 > | = 3\).

39. The magnitude of the torque is \(|r| = |r \times F| = |r||F| \sin \theta = (0.18)(60) \sin(80) \approx 10.64 \text{ N-m}\. Note that we had to convert \(r\) to meters since the force was given in Newtons.

49. (a) It does not follow! For example, consider \(a = \hat{i}, b = \hat{j},\) and \(c = \hat{k}\).

(b) It does not follow! Consider \(a = < 1, 0, 0 >, b = < 1, 0, 0 >,\) and \(c = < 2, 0, 0 >\). Since all three are parallel, cross products will be zero.

(c) Now we’re talking! With \(a \cdot b = a \cdot c,\) we have \(a \cdot (b - c) = 0\). Likewise, we have \(a \times (b - c) = 0\). Thus, \(a\) is both perpendicular to \(b - c\) and parallel to \(b - c\), so \(b - c\) must be the zero vector. Therefore, \(b = c\).