Consider the front face of each box. The curves appearing there are in the plane $x = 3$. We are looking for one of the three boxes to be $f_y$ to one of the other two (since $x$ is held constant here). Both (b) and (c) have roughly the same shape on the front face, and neither appears to be the derivative of the other. (Notice that around $y = -2$ both have slopes of zero, but neither has the value zero.) Thus (a) is $f_y$.

Now look along the right face of each box. This corresponds to the plane $y = 3$, so we are now trying to consider $f_x$. Since (b) has a roughly constant upward slope on this face but (c) is not constant, (c) cannot be $f_x$ if (b) is $f$. Thus it must be the other way around, so (c) is $f(x, y)$, (b) is $f_x(x, y)$, and (a) is $f_y(x, y)$.

10. The blue numbers indicate the $z$-values above the corresponding red curves. To estimate $f_x(2, 1)$, we hold $y$ constant at 1 and read the $z$-values from the graph near $x = 2$. We have $f_x(2, 1) \approx \frac{f(3, 1) - f(2, 1)}{3 - 2} \approx \frac{14 - 10}{1} = 4$. Also, $f_x(2, 1) \approx \frac{f(2, 1) - f(1, 1)}{2 - 1} \approx \frac{10 - 7.5}{1} = 2.5$. Averaging these gives 3.25.

Holding $x$ at a constant 2 now, we get $f_y(2, 1) \approx \frac{f(2, 2) - f(2, 1)}{2 - 1} \approx \frac{8 - 10}{1} = -2$ and $f_y(2, 1) \approx \frac{f(2, 1) - f(2, 0)}{1 - 0} \approx \frac{10 - 12}{1} = -2$. Thus it appears that $f_y(2, 1) \approx -2$.

13. $f_x = 3$, $f_y = -8y^3$.
15. $z_x = e^{3y}$, $z_y = 3xe^{3y}$.

25. $f_r = \ln(r^2 + s^2) + \frac{2r}{r^2 + s^2} = \ln(r^2 + s^2) + \frac{2r^2}{r^2 + s^2}$.

27. $u_t = e^{w/t} + t(-w/t^2)e^{w/t} = e^{w/t}(1 - w/t)$. $u_w = t(1/t)e^{w/t} = e^{w/t}$.

28. We use the Second Fundamental Theorem of Calculus: If $F(x) = \int_0^x f(t)dt$ and $f$ is appropriately continuous, then $F'(x) = f(x)$. Thus $f_x(x, y) = \cos(x^2)$. To find $f_y$, we first rewrite $f$ as $f(x, y) =$
\[- \int_x^y \cos(t^2) dt, \text{ so } f_y(x, y) = - \cos(y^2). \]

39. \( f_x(x, y) = \frac{x}{\sqrt{x^2 + y^2}}, \text{ so } f_x(3, 4) = \frac{3}{5}. \)

59. \( u_x = \frac{1}{\sqrt{x^2 + y^2}} \frac{x}{\sqrt{x^2 + y^2}} = \frac{x}{x^2 + y^2}, u_y = \frac{y}{x^2 + y^2}, u_{xy} = \frac{-2xy}{(x^2 + y^2)^2}, u_{yx} = \frac{-2xy}{(x^2 + y^2)^2}. \)

87. Since \( f_x \) and \( f_y \) are both continuous, we must have \( f_{xy} = f_{yx} \) by Clairaut’s theorem. However, we have \( f_{xy} = 4 \) and \( f_{yx} = 3 \), so this is not the case. Someone lied to us.