Solutions to Homework Assignment 16

MATH 249

Section 14.5, Page 907 Stewart 6e

1, 4, 5, 8, 10, 11, 15, 23, 25, 31, 33, 35, 40, 41, 43

5. \( w = x e^{y/z}, x = t^2, y = 1 - t, z = 1 + 2t \). \( \frac{dw}{dt} = e^{y/z}(2t) + \frac{x}{z} e^{y/z}(-1) - \frac{xy}{z^2} e^{y/z}(2) = e^{y/z} \left( 2t - \frac{x}{z} - \frac{2xy}{z^2} \right) \).

15. \( g_u(0, 0) = f_x(e^0 + \sin 0, e^0 + \cos 0) \frac{\partial x}{\partial u}(0, 0) + f_y(e^0 + \sin 0, e^0 + \cos 0) \frac{\partial y}{\partial u}(0, 0) = 2(e^0) + 5(e^0) = 7 \). \( g_v(0, 0) \) is similar.

23. \( \frac{\partial R}{\partial x} = \frac{2u}{u^2 + v^2 + w^2}(1) + \frac{2v}{u^2 + v^2 + w^2}(2) + \frac{2w}{u^2 + v^2 + w^2}(2y) \). At \( (1, 1) \), \( u = 3, v = 1, w = 2 \), so we get \( \frac{1}{14}(6 + 4 + 8) = \frac{9}{7} \). The other part is similar.

31. Let \( F(x, y, z) = x^2 + y^2 + z^2 - 3xyz \). If we set \( F(x, y, z) = 0 \), we find that \( \frac{\partial z}{\partial x} = -\frac{2x - 3yz}{2z - 3xy} \) and \( \frac{\partial z}{\partial y} = -\frac{2y - 3xz}{2z - 3xy} \).

35. \( \frac{dT}{dt} = \frac{\partial T}{\partial x} \frac{dx}{dt} + \frac{\partial T}{\partial y} \frac{dy}{dt} = T_x \frac{1}{\sqrt{1 + t}} + T_y \cdot \frac{1}{3} \). At \( t = 3 \), \( (x, y) = (2, 3) \), so we have \( T'(3) = 4 \frac{1}{4} + 3 \cdot \frac{1}{3} = 2 \) degrees Celsius per second. Pretty fast!

41. \( V = \frac{8.31T}{P} \) according to Example 2. \( \frac{dV}{dt} = \frac{8.31}{P} T'(t) - \frac{8.31T}{P^2} P'(t) \). \( T' \) and \( P' \) are given to us as constants, so we can compute \( V' = 8.31(0.15/20 - 320(0.05)/20^2) \approx -0.27 \) liters per second.

43. \( \frac{\partial z}{\partial r} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial r} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial r} = z_x \cos \theta + z_y \sin \theta \). Also, \( \frac{\partial z}{\partial \theta} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial \theta} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial \theta} = -z_x r \sin \theta + z_y r \cos \theta \). It is not hard to see now that \( \left( \frac{\partial z}{\partial r} \right)^2 + \frac{1}{r^2} \left( \frac{\partial z}{\partial \theta} \right)^2 = \left( \frac{\partial z}{\partial x} \right)^2 + \left( \frac{\partial z}{\partial y} \right)^2 \).