Maple Funsheet 4: Level Curves and Limits

MATH 249

Directions: By the end of the day on Monday, February 17, e-mail me and all of your group members your Maple worksheet as an attachment with all output deleted. I will return my comments by replying. Please turn in a group worksheet (at least two people per group and at most four), but be sure to enter all group members’ names at the beginning. Also, I do not want to see all of your scratch work: turn in as clean a Maple file as possible. (I do want to see your commands, though.)

Work together on each problem; do not delegate different problems to different people.

To graph level curves of a function, you first need to load the Student[MultivariateCalculus] package:

\[ \text{with(Student[MultivariateCalculus])}; \]

Once the package is loaded, use

\[ \text{CrossSection}(f(x,y),z=[L_1,L_2,L_3,\ldots,L_n],x=a..b,y=c..d,axes=boxed); \]

to plot the surface along with the level curves corresponding to \( z = L_1, z = L_2, \ldots, z = L_n \).

1. Use Maple to graph each function and plot the appropriate level curves. Identify the level curves where possible. (E.g., are they circles? Parabolas? Something else identifiable?)
   (a) \( f(x, y) = x^2 + 4y^2, z = 0, 2, 4, 6, 8 \).
   (b) \( f(x, y) = \frac{5 \sin(xy)}{e^{xy} + 1}, z = 0, 1, 2, 3 \).

2. Use Maple to compute each limit. The syntax is \( \text{limit}(f(x,y),\{x=a,y=b\}) \); Also graph the function in question to see whether the result makes sense. (If Maple returns nothing, it means Maple doesn’t know how to deal with the limit.)
   (a) \( \lim_{(x,y) \to (0,0)} \frac{x^2 - y^2}{x^2 + y^2} \)
   (b) \( \lim_{(x,y) \to (0,0)} \frac{2x^2 + 3xy + 4y^2}{3x^2 + 5y^2} \)
   (c) \( \lim_{(x,y) \to (0,0)} \frac{x^2 - y^2}{x + y} \)
   (d) \( \lim_{(x,y) \to (0,0)} \frac{xy^3}{x^2 + y^6} \)

It is also possible to specify a particular path of approach by using \( y = f(x) \) or \( x = f(y) \) in place of \( \{x=a,y=b\} \), and then, in a separate \( \text{limit} \) command, letting the remaining variable go to wherever it is supposed to. (Example: \( \text{limit(limit(x^2+2*x*y, \{y=x^2\}), \{x=2\})} \); will approach the point (2,4) along the path \( y = x^2 \).)

3. For each limit in number 2 that does not exist, find two paths that give different results.