1. Let $V = \{(x, x, 4x) | x \in \mathbb{R}\}$.

   (a) Prove that $V$ is closed under addition.

   (b) Prove that $V$ is closed under scalar multiplication.

   (c) Is $V$ a subspace of $\mathbb{R}^3$?

**Solution:**

(a) Let $u, v \in V$. Then $u = (x, x, 4x)$ and $v = (y, y, 4y)$ for some $x, y \in \mathbb{R}$. Now $u + v = (x, x, 4x) + (y, y, 4y) = (x + y, x + y, 4x + 4y) = (x + y, x + y, 4(x + y)) \in V$. Therefore, $V$ is closed under addition.

(b) Let $u$ be as above, and let $r \in \mathbb{R}$. Then $ru = r(x, x, 4x) = (rx, rx, 4rx) \in V$.

(c) Yes; since $V$ is nonempty and closed under addition and scalar multiplication, $V$ is a subspace of $\mathbb{R}^3$.

2. Let $V = \{(2x + 1, x) | x \in \mathbb{R}\}$.

   (a) Prove that $V$ is not closed under addition.

   (b) Prove that $V$ is not closed under scalar multiplication.

   (c) Is $V$ a subspace of $\mathbb{R}^3$?

**Solution:**

(a) Let $x = 0$. Then $(2x + 1, x) + (2x + 1, x) = (1, 0) + (1, 0) = (2, 0) \notin V$ since $2 \neq 2(0) + 1$.

(b) With $x = 0$ and $r = 2$, we have $r(2x + 1, x) = 2(1, 0) = (2, 0) \notin V$ as in (a).

(c) No; $V$ is not closed under addition, so it cannot be a subspace of $\mathbb{R}^3$,