Solutions to Homework Assignment 15

MATH 345-01
Section 28, Page 89
1,2,8,10

1. (a) \(e^{2 \pm 3\pi i} = e^2 (\cos 3\pi \pm i \sin 3\pi) = -e^2\).
   (b) \(e^{(2+\pi i)/4} = e^{1/2} (\cos \pi/4 + i \sin \pi/4) = \frac{\sqrt{2} \sqrt{e}}{2} (1 + i) = \sqrt{2} e^{1/2}(1 + i)\).
   (c) \(e^{z+\pi i} = e^z e^{\pi i} = -e^z\).

2. \(2, z^2, 3, z, e^z, \text{ and } e^{-z}\) are all entire, so any sum of products of them is also entire by theorems in section 19.

8. (a) \(z = \log -2 = \ln |-2| + i \arg(-2) + \ln 2 + i(\pi + 2k\pi)\).
   (b) \(z = \log(1 + \sqrt{3}i) = \ln(2) + i \arg(1 + \sqrt{3}i) = \ln 2 + i(\pi/3 + 2k\pi)\).
   (c) \(2z - 1 = \log 1 = i(2k\pi), \text{ so } z = k\pi i + \frac{1}{2}\).

10. (a) Suppose that \(e^z = e^x \cos x + i \sin y\) is real. Then \(\sin y = 0\), so \(y = k\pi\) for some integer \(k\). Since \(y = \Im z\), we have \(\Im z = k\pi\).
   (b) Suppose that \(e^z = e^x \cos x + i \sin y\) is pure imaginary. Then \(\cos y = 0\), so \(y = (2k + 1)\pi/2\) for some integer \(k\). Since \(y = \Im z\), we have \(\Im z = (2k + 1)\pi/2\).