Solutions to Homework Assignment 18

MATH 345-01
Section 32, Page 99
1,2,3,5,7,8bc,9

1. (a) \((1 + i)^i = e^{i \log(1 + i)} = e^{i \ln \sqrt{2} - \pi/4 + 2\pi n} = e^{-\pi/4 + 2\pi n} e^{i(\ln 2)/2}\).

(b) \((-1)^{1/\pi} = e^{(1\pi) \log(-1)} = e^{(1/\pi)(i\pi + 2\pi n)} = e^{i(2n + 1)}\).

2. (a) We did this in class.

(b) \(\left(\frac{e}{2}(-1 - \sqrt{3}i)\right)^{3\pi i} = \exp(3\pi i (\ln e - i2\pi/3)) = \exp(3\pi i - 2\pi^2) = -e^{2\pi^2}\).

(c) \((1 - i)^4i = e^{4i(\ln \sqrt{2} - \pi i/4)} = e^{\pi/2} e^{2i \ln 2}\).

3. \((-1 + \sqrt{3}i)^{3/2} = e^{(3/2)(\ln 2 + 2\pi i/3)} = e^{3\ln 2/2} e^{\pi i} = -e^{(\ln 8)/2} = -8^{1/2} = \pm 2\sqrt{2}\).

5. For the principal root, we have \(z_1^{1/n} = e^{(1/n)\text{Log} z_0} = e^{(1/n)(\ln |z_0| + i\text{Arg} z_0)} = \sqrt[n]{|z_0|} e^{i\Theta/n}.\) From Section 8, we have \(z_0^{1/n} = \sqrt[n]{|z_0|} e^{i\Theta/n + 2k\pi/n},\) and taking the principal value of this gives the desired expression.

7. \(|e^c\log z| = |e^{(a+bi)(1\pi/2+i2k\pi)}| = |e^{-b\pi/2-2k\pi} e^{a(i\pi/2+i2k\pi)}| = e^{-b\pi/2-2k\pi}.\) This varies with \(k\) unless \(b = 0,\) so we require that \(c\) be real.

8. (b) We have \((e^z)^n = (e^{\text{Log} z})^n = e^{nc\text{Log} z}\) from earlier work \(((e^z)^n = e^{nz}).\) Thus we get \(z^{nc},\) as desired.

(c) \(z^c z^d = e^{c\text{Log} z} e^{d\text{Log} z} = e^{(c+d)\text{Log} z} = z^{c+d}\).

9. \(\frac{d}{dz} e^{f(z)} = \frac{d}{dz} e^{f(z)} \log c = e^{f(z)} \log c f'(z) \log c = e^{f(z)} f'(z) \log c.\)