Throughout, all variables represent integers unless otherwise specified.

1. Find the continued fraction expansion of $\frac{235}{71}$.

**Solution:**

\[
235 = 71(3) + 22 \\
71 = 22(3) + 5 \\
22 = 5(4) + 2 \\
5 = 2(2) + 1 \\
2 = 1(2) + 0.
\]

Thus

\[
\frac{235}{71} = 3 + \frac{1}{3 + \frac{1}{4 + \frac{1}{2 + \frac{1}{2}}}}.
\]

2. Find the value of the infinite continued fraction $[2; 1, 2, 1, 2, 1, 2, ...] = [2; \overline{1, 2}]$.

**Solution:** Let $x = [2; \overline{1, 2}]$. Note that $x - 2 = \frac{1}{x + 1} = \frac{x}{x + 1}$, so $(x - 2)(x + 1) = x$.

Thus $x^2 - 2x - 2 = 0$. Therefore, $x = \frac{2 \pm \sqrt{12}}{2} = 1 \pm \sqrt{3}$. Only the positive root gives a positive value, so $x = 1 + \sqrt{3}$.

3. Find the infinite continued fraction expansion of $\sqrt{10}$.

**Solution:** Since $\sqrt{10} = 3$, our expansion begins with 3. We have $\sqrt{10} = 3 + \frac{1}{\sqrt{10} - 3} = 3 + \frac{1}{\sqrt{10} - 3}$. Rationalizing the denominator (of the denominator!) gives

\[
\sqrt{10} = 3 + \frac{1}{3 + \sqrt{10}} = 3 + \frac{1}{6 + (\sqrt{10} - 3)},
\]

and we see that we are already repeating our calculations, so $\sqrt{10} = [3; 6]$.

4. Find the infinite continued fraction expansion of $\sqrt{13}$.

**Solution:**

\[
\sqrt{13} = 3 + \frac{1}{\frac{1}{\sqrt{13} - 3} \cdot \frac{\sqrt{13} + 3}{\sqrt{13} + 3}} = 3 + \frac{1}{\sqrt{13} + 3}
\]

\[
= 3 + \frac{1}{4 + \frac{\sqrt{13} - 1}{4}}. \text{ Observe that } \frac{\sqrt{13} + 3}{4} = 1 + \frac{\sqrt{13} - 1}{4}.
\]
etc. It ends up as $[3; \overline{1,1,1,1,6}]$.

5. Let $x$ be an irrational number with convergents $p_n/q_n$. Show that for every $n \geq 0$,

$$\left| x - \frac{p_n}{q_n} \right| < \left| x - \frac{p_{n-1}}{q_{n-1}} \right|.$$