Proposition: (Theorem 0.4.6) Let \((n_i)\) be a strictly increasing sequence of natural numbers. Then \(i \leq n_i\), for all \(i \in \mathbb{N}\).

Proof (Induction) Let \((n_i)\) be a strictly increasing sequence of natural numbers.

Base Case: When \(i = 1\), \(n_i \geq 1 = i\) because \((n_i)\) is a sequence of natural numbers.

Induction Hypothesis: Let \(i \leq n_i\) for all \(i \in \mathbb{N}\). We will show \(i + 1 \leq n_{i+1}\).

We have from the base case that \(n_i \geq i\). It follows that \(n_i + 1 \geq i + 1\). Recall, by the definition of a strictly increasing sequence if \(j < m\) then \(n_j < n_m\) for all \(j, m \in \mathbb{N}\). From this definition we know that \(i + 1 \geq i\). Seeing as how this is a sequence of natural numbers, the smallest increase in value would be +1, we can specify that \(n_{i+1} - n_i \geq 1\). Considering \(n_i + 1 \geq i + 1\) and \(n_{i+1} \geq n_i + 1\), Thus, by transitivity \(n_{i+1} \geq i + 1\). ■