Problem 0.4.4 part b

**Theorem 1.** Prove that either \((s_i)\) has a constant subsequence or a distinct subsequence.

**Proof.** Let \((s_i)\) be a sequence \(\in A\). Suppose that \((s_i)\) doesn’t have a constant subsequence. Then by negating part a we have that for all \(a \in A\), there exists \(n \in \mathbb{N}\) where there is not a \(j \geq n\) such that \(s_j = a\). So if \(s_j = a_1\) where \(j < n\), then when \(i > n\), \(s_i \neq a_1\) where \(s_i \neq s_j\) for distinct \(i, j \in \mathbb{N}\). We will let \(a_2 = s_i\), then for some \(n_2\) such that \(i < n_2\) and if \(l > n_2\), we know \(s_l \neq a_2, a_1\). Suppose that \(s_{i_1} \ldots s_{i_k}\) are distinct and \(i_1 < i_2 < \ldots\) if \(s_{i_k} = a_k\) then there exists an \(n\), where \(i > n\), such that \(s_i \neq s_{i_1} \ldots s_{i_k}\). Let \(a_{k+1} = s_{n+1}\), then \(s_{n+1} \neq s_{i_1} \ldots s_{i_k}\). Then \(a_1, \ldots, a_k, a_{k+1}, \ldots\) is a sequence of distinct terms. \(\square\)