Problem 1.3.10

(2) Proposition: Let $a$ be a real number. Show $|a| \geq 0$.

Proof.

Let $a \in \mathbb{R}$.

Case 1: Suppose $a \geq 0$. By definition, it follows that $|a| = a$. Since $a \geq 0$, this implies that $|a| \geq 0$ as well.

Case 2: Suppose $a < 0$. By definition, it follows that $|a| = -a$ and $0-a \in \mathbb{R}^+$. Since $0-a = 0+(-a) = -a$ by definition of additive identity, it follows that $-a \in \mathbb{R}^+$. Further, since $|a| = -a$, this implies that $|a| \in \mathbb{R}^+$. Thus, $|a| > 0$.

Therefore, when $a \in \mathbb{R}$, it follows that $|a| \geq 0$. ■