Prove Theorem 3.1.7 2 → 3

Let \((X, d)\) be a metric space. Let \(U\) be a subset of \(X\).

Given for every \(a \in U\), there exists \(r > 0\) such that \(B_r(a) \subseteq U\).

\textbf{Proof} \(\text{Let } a \in U. \text{ From given } B_r \subseteq U. \text{ By definition of open ball } B_r(a) = \{z \in X : d(a, z) < r\}.\)

By symmetry \(d(a, z) = d(z, a)\). Let \(x \in X\) such that \(d(a, x) = d(x, a) < r\), also by symmetry.

Therefore \(x \in B_r(a)\). Since \(B_r(a) \subseteq U\), \(x \in U\).

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