1. Proof of theorem 3.1.7, 3 → 1

Proof. Suppose for every \( a \in U \) there exists \( r > 0 \) such that if \( d(x, a) < r \), then \( x \in U \). Let \( m \in U \), and let \( r_1 > 0 \) be the real number such that if \( d(x, m) < r \), then \( x \in U \). It follows by the definition of open balls that \( m \in B_{r_1}(m) \subset U \). Thus every element of \( U \) is contained in an open ball that is a subset of \( U \). It follows that \( U \) is the union of these open balls, and thus by the definition of open, \( U \) is open. \( \square \)