Let $X$ be a metric space with $S \subset X$. The following are equivalent:

(1) $S$ has finite diameter.

(2) There exists $a \in X$ and $r > 0$ such that $S \subset B_r(a)$.

(3) For all $a \in X$, there is $r > 0$ such that $S \subset B_r(a)$.

(1) implies (2): Assume $S$ has finite diameter. Then there is a $diam(S) = \sup \{d(a, b) : a, b \in S\}$. So choose $r$ such that $r > diam(S) > 0$. Since $a, b \in S$ and $s \subset X, a, b \in X$. By the definition of open balls, we can construct $B^X_r(a)$. Then for all $n \in S, d(a, n) \leq diam(S)$ by the definition of diameter. Thus, $d(a, n) < r$.

Since $n \in S$, and $S \subset X, n \in X$. So by theorem 3.1.73, $n \in B^X_r(a)$. Therefore, $S \subset B^X_r(a)$. 

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