1. 3.6.3 Prove a set $C$ is closed iff if $(a_n)$ is a sequence in $C$ converging to $x$, then $x \in C$.

Proof. Let $C$ be a closed set, and let $(a_n)$ be a sequence in $C$ converging to $x$. We break this into two cases. In the first case, suppose that $x \in (a_n)$. Since $a_n \in C$ for all $a_n \in (a_n)$, it follows that $x \in C$. Now suppose that $x \notin (a_n)$. It follows then that there exists a sequence of points in $C - \{x\}$ that converge to $x$. Therefore, by theorem 3.5.1, $x$ is a limit point of $C$. Since $C$ is closed, it follows that $x \in C$. Now suppose that if $(a_n) \in C$, and $(a_n) \to x$, then $x \in C$. Let $x$ be a limit point of $C$. It follows that there exists a sequence of distinct points in $C$ that converge to $x$. Therefore, there is a sequence in $C$ that converges to $x$, so by hypothesis, $x \in C$. It follows that $C$ contains all of its limit points, and therefore is closed.