prop: Let \((X, d_X)\) and \((Y, d_Y)\) be metric spaces, \(f : X \rightarrow Y\) be a function, and \(a \in X\). If given any sequence \((x_n)\) in \(X\) converging to \(a\), \((f(x_n))\) converges to \(f(a)\), then \(f\) is continuous at \(a\).

**Proof**

Suppose for all \((x_n) \in X\) such that \(x_n \rightarrow a\), \(f(x_n) \rightarrow f(a)\). Since this is true for all sequences, it must be true for all distinct sequences. By Theorem 4.2.4, \(\lim_{x \rightarrow a} f(x) = f(a)\). By Definition 4.3.1, \(f\) is continuous at \(a\). ■