Proposition: Theorem 4.4.4 Prove that Lipschitz functions are uniformly continuous.

Proof Let $f : X \to Y$ be a Lipschitz function. Therefore for some $k > 0$ for all $a, b \in X$, $d_Y(f(a), f(b)) \leq kd_X(a, b)$. Let $\epsilon > 0$. Note that then $\frac{\epsilon}{k} > 0$ and therefore let $\delta$ be such that $0 < \delta < \frac{\epsilon}{k}$. Suppose $d_X(a, b) < \delta$. Consider $d_Y(f(a), f(b))$. By definition of Lipschitz functions $d_Y(f(a), f(b)) \leq kd_X(a, b) < k\delta < k\frac{\epsilon}{k} = \epsilon$. Therefore by definition the function is uniformly convergent.