Problem 9.2.8a

**Theorem 1.** Prove Theorem 9.2.6.
a) Use Theorem 9.2.2 to prove that the constant multiple rule holds.
If \( k \in \mathbb{R} \), the function \( kf \) is differentiable at \( x \) and \( (kf)'(x) = kf'(x) \).

*Proof.* Theorem 9.2.2 allows us to write

\[
 kf(y) = kf'(x)(y - x) + rf(x) + r(y) \]

which implies

\[
 kf(y) = k[f'(x)(y - x) + rf(x) + kr(y)]
 = kf'(x)(y - x) + kf(x) + kr(y)
\]

Since \( \lim_{y \to x} \frac{r(y)}{y - x} = 0 \), then \( \lim_{y \to x} \frac{kr(y)}{y - x} = 0 \) because \( \lim_{y \to x} \frac{kr(y)}{y - x} = k \lim_{y \to x} \frac{r(y)}{y - x} = k0 = 0 \) by Part 4 of Theorem 3.4.11. It follows by Theorem 9.2.2 \( kf \) is differentiable. Also note: \( (kf)' \) happens at \( kf'(x) \) so \( (kf')(x) = kf'(x) \).