Let $f : \mathbb{R} \to \mathbb{R}$ be a function. Show that $\lim_{x \to a} f(x)$ exists.

e) $f(x) = \frac{1}{x-2}$, $a = 2$. From this we can see that the limit should be $L = -\frac{1}{2}$.

\[\text{Proof}\]
Let $\epsilon > 0$. Choose $\delta = \min(1, 2\epsilon)$. Suppose $0 < |x - 2| < \delta$. Note that $x \in (1, 3)$.

Thus, $d(f(x), L) = \left| \frac{1}{x-2} + \frac{1}{2} \right| = \frac{2 + (x-4)}{2(x-4)} = \frac{|x-2|}{2|x-4|} < \frac{1}{2}(2\epsilon) = \epsilon$. So by definition of a limit, $\lim_{x \to 2} f(x) = -\frac{1}{2}$. \blackbox