Theorem 3.4.11.1

Proof

Let \((a_n)\) and \((b_n)\) be sequences in \(\mathbb{R}\). And let \(k\) be any real number. Assume \(a_n \to L\) and \(b_n \to M\). Now consider that \(|(a_n + b_n) - (L + M)| = |(a_n - L) + (b_n - M)|\). And then by the triangle inequality it follows that \(|(a_n - L) + (b_n - M)| \leq |a_n - L| + |b_n - M|\). Where \(\varepsilon > 0\) consider \(\varepsilon/2\). Now by definition of convergence we know for each \(\varepsilon/2 > 0\) that \(\exists N_1 \in \mathbb{N}\) such that \(\forall n > N_1\) that \(|a_n - L| < \varepsilon/2\). And again we know for each \(\varepsilon/2 > 0\) that \(\exists N_2 \in \mathbb{N}\) such that \(\forall n > N_2\) that \(|b_n - M| < \varepsilon/2\). Now let \(N = \max(N_1, N_2)\) where \(n > N\). Then it follows that \(|a_n - L| + |b_n - M| < \varepsilon/2 + \varepsilon/2 = \varepsilon\). Thus by our first consideration we know that \(|(a_n + b_n) - (L + M)| < \varepsilon\). And therefore by the definition of convergence we know that \((a_n + b_n) \to (L + M)\).