Prove that \((s_i)\) has a constant subsequence if and only if there exists \(a \in A\) such that for all \(n \in \mathbb{N}\) there exists \(j \geq n\) such that \(s_j = a\).

\((\Rightarrow)\) Assume \((s_i)\) has a constant subsequence. So, \((s_{n_i})\) is constant, and there exists \(a \in A\) such that \(s_{n_i} = a\) for all \(i \in \mathbb{N}\). Let \(m \in \mathbb{N}\). Since \(n_i\) is strictly increasing, \(m \leq n_m\) by theorem 0.4.6, so \(s_{n_m} = a\).