Problem 1.3.8 part a (Theorem 1.3.6 part a)

Prove: For $a, b, c, d \in \mathbb{R}$, if $a > b$ and $c \geq d$, then $a + c > b + d$.

Let $a, b, c, d \in \mathbb{R}$, and suppose $a > b$ and $c \geq d$. By Definition 1.3.2, $a > b \Rightarrow a - b \in \mathbb{R}^+$ and $c \geq d \Rightarrow c - d \in \mathbb{R}^+ \cup \{0\}$. Then by the Order Axiom, since $\mathbb{R}^+$ is closed under addition, $(a - b) + (c - d) \in \mathbb{R}^+$. It is important to note that for the case where $c - d = 0$, $(a - b) + (c - d) = (a - b) + 0 = a - b \in \mathbb{R}^+$ (Additive Identity), and so $(a - b) + (c - d) \in \mathbb{R}^+$ for all cases. And so,

\[
(a - b) + (c - d) \in \mathbb{R}^+ \\
(a + c) + (-b - d) \in \mathbb{R}^+ \quad \text{Commutativity and Associativity} \\
(a + c) + ((-1)(b + d)) \in \mathbb{R}^+ \quad \text{By Field Axiom of Distribution} \\
(a + c) - (b + d) \in \mathbb{R}^+ \quad \text{By Problem 1.2.3} \\
(a + c) > (b + d) \quad \text{By Definition 1.3.2}
\]

$\square$