P1.4.8a

Proposition: For any $x \in \mathbb{R}$ there is an integer $n$ such that $n - 1 \leq x < n$.

Proof: Let $S = \{ z \in \mathbb{Z} | z > x \}$. By definition, it is clear that $S$ is bounded below, at least by $x$. Also, by Theorem 1.4.6, there exists some natural number greater than $x$, so $S$ is non-empty. By Theorem 0.3.4, $S$ must have a least element, call this element $n$. By definition of least element, $n - 1 \notin S$, so from our definition of $S$, $n - 1 \not> x$, so $n - 1 \leq x < n$.

$\blacksquare$