Problem 1.4.8 (b) For all $x \in \mathbb{R}$ and $N \in \mathbb{N}$, there exists an $n \in \mathbb{N}$ such that $\frac{n-1}{N} \leq x \leq \frac{n}{N}$.

Proof: since $x, N \in \mathbb{R}$, then $xN \in \mathbb{R}$ by the field axioms. By part (a), there exists $n \in \mathbb{N}$ that satisfies $n-1 \leq xN \leq n$. We can divide all parts of this inequality by $N$ (which is valid because $N \neq 0$) to yield $\frac{n-1}{N} \leq \frac{xN}{N} = x \leq \frac{n}{N}$ as desired. QED