P.2.2.1 Let \((X,d)\) be a metric space. Let \(x, y \in X\). Suppose that for all positive real numbers \(\epsilon\), \(d(x,y) \leq \epsilon\). Prove that \(x = y\).

Since \((X,d)\) is a metric, \(d(x,y) \geq 0\) for all \(x, y \in X\). Thus part a of P.1.4.2 can be applied to show that \(d(x,y) = 0\). The positive definite property thereby asserts that \(x = y\). \(\Delta\)