Problem 3.1.10

Jeff Schreiner-McGraw

February 17, 2013

Part b!

Proof. First, assume that $S$ is bounded. By theorem 3.1.12, there is an $r > 0$ such that $S \subset B_r(0)$. Now let $s \in S$. We know that $|s - 0| < r$. Therefore, there exists $K = r$ such that $|s| < K$ for all $s \in S$.

Now, assume that there exists $K \in \mathbb{R}$ such that $|s| \leq K$ for all $s \in S$. To satisfy part two of Theorem 3.1.12, we have $a = 0$ and $r = K + 1 > 0$. Consider $B_r(a)$. Take arbitrary $s \in S$. We know that $d(x,0) = |s - 0| = |s| \leq K < r$. So, $s \in B_r(a)$ and thus $S \subset B_r(a)$. \qed