Proposition: Let $X$ be a metric space with subset $S$. If there exists some $a \in X$ and $r > 0$ such that $S \subseteq B_r(a)$, then for all $x \in X$, there exists some $q > 0$ such that $S \subseteq B_q(x)$.

**Proof:** Let $X$ be a metric space with distance function $d$ and subset $S$, and suppose that there exists some $a \in X$ and $r > 0$ such that $S \subseteq B_r(a)$. Let $x \in X$, and let $q = d(a,x) + r$. We claim that $S \subseteq B_q(x)$. Let $s \in S$, so by hypothesis, $s \in B_r(a)$, so $d(a,s) < r$, by definition of open balls. Consider $d(x,s)$, which is at most $d(x,a) + d(a,s)$, by the triangle inequality. Thus, by transitivity of inequality, we have $d(x,s) < d(x,a) + r$, so $d(x,s) < q$, by definition of $q$, so it follows that $s \in B_q(x)$, and as such $S \subseteq B_q(x)$. Thus, if there exists some $a \in X$ and $r > 0$ such that $S \subseteq B_r(a)$, then for all $x \in X$, there exists some $q > 0$ such that $S \subseteq B_q(x)$. 

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