Problem 6.2.1 Lipschitz functions are uniformly continuous.

Proof:

Let $f : X \to Y$ be a function between metric spaces that satisfies the Lipschitz condition with constant $k > 0$. Let $\varepsilon > 0$ and $\delta = \frac{\varepsilon}{k}$. Let $a, b \in X$ where $d_X(a, b) < \delta$. Then $d_Y(f(a), f(b)) \leq k d_X(a, b) < k \frac{\varepsilon}{k} = \varepsilon$. We conclude $f$ is uniformly continuous.