Problem D.2.1 (b)

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February 25, 2013

Prove whether or not the following sequence of real numbers converges.

(b) $1, -1, 1, -1, 1, -1, 1, -1, \ldots$

Let $(a_n)$ be the above sequence. Observe that if $n$ is odd, then $a_n = 1$, and otherwise $a_n = -1$. Suppose that $(a_n)$ converges to some real number $a$. Then let $\epsilon = \frac{1}{2}$.

By definition of convergence, there exists some natural number $N$ such that for all natural numbers $n > N$, $d(a, a_n) < \epsilon$.

We can find some $i, j > N$ such that $a_i = 1$ and $a_j = -1$. So $d(1, a) < \epsilon$ and $d(-1, a) < \epsilon$. Thus, $d(1, a) + d(a, -1) < 2\epsilon$.

By the triangle inequality, $d(1, -1) < 2\epsilon$. But $d(1, -1) = |1 - (-1)| = |2| = 2$.

So $2 < 2\epsilon$. Since $\epsilon = \frac{1}{2}$, $2 < 1$. However, we know that 2 is greater than 1. This is a contradiction, so $(a_n)$ does not converge.