Problem E2.1(d)

Show that \( \lim_{x \to 2} (x^4 + x + 2) = 20 \).

Let \( \epsilon > 0 \). Pick \( \delta \) such that \( \delta = \min(1, \frac{\epsilon}{66}) \). Now suppose \( 0 < |x - 2| < \delta \). We know that \( \delta \leq 1 \) and it follows that \( x \in (1, 3) \). So, then we know that the distance function

\[
\begin{align*}
    d(f(x), L) &= |x^4 + x + 2 - 20| = |x^4 + x - 18| \\
    &= |x - 2||x^3 + 2x^2 + 4x + 9| \\
    &\leq |x - 2|[27 + 18 + 12 + 9] \\
    &< \frac{\epsilon}{66}(66) = \epsilon
\end{align*}
\]

Therefore, \( \lim_{x \to 2} (x^4 + x + 2) = 20 \).