1. Since every group element has an inverse, \( c \) has an inverse \( d \). Thus \( c^2 = c \) implies \( d(c^2) = dc \), which in turn implies \( (dc)c = dc \). Thus \( ec = e \), so \( c = e \).

2. \( a = (132), b = (23) \). Thus \( ab = (13) \), so \( (ab)^{-1} = ab \). On the other hand, \( a^{-1}b^{-1} = (123)(23) = (12) \neq (ab)^{-1} \).

3. \((abcd)^{-1} = d^{-1}c^{-1}b^{-1}a^{-1}\).

6. In \( S_3 \), \((12)^2 = (13)^2 = (23)^2 = c^2 = e \). We have \textbf{four} solutions!

7. \( n = 5, 5, 2^1 = 1, \) so \( o(5) = 2 \).

11. \( x = a^{-1}b \) is a solution for \( ax = b \) and \( y = ba^{-1} \) is a solution for \( ya = b \). If \( x' \) is another solution to \( ax = b \), then \( ax = ax' \implies x = x' \). Similarly, if \( y' \) is a second solution for \( ya = b \), then \( ya = y'a \implies y = y' \). Thus, in both cases, the solution is unique.

15. \( a = (132) \) and \( b = (13) \), \( x = (23) \) is the solution to \( ax = b \) from Exercise 2. To get \( ya = b \), we need \( y = ba^{-1} = (13)(123) = (12) \neq x \).

17. Note that \((bab^{-1})^n = (bab^{-1})(bab^{-1}) \cdots (bab^{-1}) = ba^nb^{-1}\). If \( o(a) = n \), then \((bab^{-1})^n = ba^nb^{-1} = beb^{-1} = e \), so \( o(bab^{-1}) \leq n \). On the other hand, if \( m = o(bab^{-1}) \), then \((bab^{-1})^m = ba^mb^{-1} = e \), so \( ba^m = b \) and \( a^m = e \). Thus \( m \geq n \). Therefore, \( o(bab^{-1}) = o(a) \).

22. Let \( a, b \in G \). We must show that \( ab = ba^{-1} \). Let \( c = aba^{-1} \). Then \( ca = aba^{-1}a = ab \), so by assumption, \( b = c \). Thus \( ab = ba \).

28. Let \( n = o(ab), m = o(ba) \). Then \((ba)^{n+1} = (ba)(ba) \cdots (ba) = b(ab)(ab) \cdots (ab)a = ab(ab)^na = bea = ba \). That is, \((ba)^{n+1} = ba \), so \((ba)^n = e \). Therefore, \( m \leq n \). Reversing the roles of \( a \) and \( b \) shows that \( n \leq m \), as well, so \( m = n \).

30. Note first that the elements of order two are precisely the non-identity elements that are their own inverse. In any group, every element has an inverse. Let \( n \) be the number of non-identity elements that are self-inverse, and let \( b_1, \ldots, b_k \) have inverses \( c_1, \ldots, c_k \), respectively, where \( \{b_1, \ldots, b_k\} \) and \( \{c_1, \ldots, c_k\} \) are disjoint sets. Then \( |G| = 1 + n + 2k \). (The 1 is for the identity element.) Since \( |G| \) is even, \( n \geq 1 \), so some element has order two.

33. If we can show that \( b^3 = e \), it will follow immediately that \( ab = ba \) since we already know that \( ab = b^4a \).

Now \( ab = b^4a \implies b^2ab = b^6a = a \). Now \((b^2ab)b^5 = ab^7\), so \( b^2a = ab^5 = ab(b^4) = (b^4a)b^4 = b^4(ab)b^3 = b^4(b^4a)b^3 = b^6ab^3 = b^2ab^3 \). Thus \( e = b^3 \).