1. Since $K$ is separable over $F$, each element of $K$ is separable and therefore algebraic over $F$. Thus $K$ is also algebraic over $E$. Now let $u \in K$. Since $K$ is separable over $F$, the minimal polynomial $p(x) \in F[x]$ of $u$ is separable. But $p(x) \in E[x]$, too, so $p(x)$ remains separable. Therefore, $K$ is separable over $E$.

2. Since $1_K = 1_F$, $n \cdot 1_K = 0$ implies $n \cdot 1_F = 0$, which is impossible for $n > 0$ since $F$ has characteristic zero.

3. Let $F$ be a field of characteristic zero. The elements $n \cdot 1_F$ are distinct for $n \in \mathbb{Z}^+$ since otherwise $n \cdot 1_F = m \cdot 1_F$ for some $m < n$, which implies that $(n - m) \cdot 1_F = 0$ and thereby contradicts the assumption that $F$ has characteristic zero. Therefore, $F$ must be infinite.

8. Suppose that $u$ is a repeated root of $f(x)$. Then $f(x) = (x - u)^2g(x)$ for some $g(x) \in K[x]$, so $f'(x) = 2(x - u)g(x) + (x - u)^2g'(x)$, which implies that $u$ is also a root of $f'(x)$. Conversely, assume that $u$ is a root of both $f(x)$ and $f'(x)$. Then $f(x) = (x - u)g(x)$, so $f'(x) = g(x) + (x - u)g'(x)$ and also $f'(x) = (x - u)h(x)$ for some $h(x) \in K(x)$. Since the expressions for $f'(x)$ must be equal, this implies that $g(u) = 0$, so $g(x) = (x - u)g_1(x)$ for some $g_1(x) \in K[x]$. Thus $f(x) = (x - u)^2g_1(x)$, and $u$ is a repeated root of $f(x)$.

9. If $f(x)$ and $f'(x)$ are relatively prime, then Lemma 10.6 tells us that $f(x)$ is separable. Assume now that $f(x)$ is separable. Then no root of $f(x)$ is repeated, so $f(x)$ and $f'(x)$ share no common roots by Exercise 8. If $h(x)$ is a common factor, then any root of $h(x)$ is a root of both $f(x)$ and $f'(x)$, so $h(x)$ cannot have any roots. That is, any common factor of $f(x)$ and $f'(x)$ must be constant, so they are relatively prime.

12. (a) We have $v = \sqrt{2}$ and $w = \sqrt{3}$. Also, $v_2 = -\sqrt{2}$ and $w_2 = -\sqrt{3}$ (the other roots of the minimal polynomials of $\sqrt{2}$ and $\sqrt{3}$). We need $c \neq \frac{\sqrt{2} - \sqrt{2}}{\sqrt{3} - (-\sqrt{3})} = \frac{\sqrt{2} - \sqrt{2}}{\sqrt{3} - (-\sqrt{3})}$, which is to say, $c \neq 0, -\sqrt{2}/3$. I will choose $c = 1$, giving $Q(\sqrt{2}, \sqrt{3}) = Q(\sqrt{2} + \sqrt{3})$.

(b) Now we require $c \neq 0, -\sqrt{3}/i$. Again let $c = 1$ to get $Q(\sqrt{3}, i) = Q(\sqrt{3} + i)$.

c) Use part (a) and $c = 1$ to get $Q(\sqrt{2}, \sqrt{3}, \sqrt{5}) = Q(\sqrt{2} + \sqrt{3} + \sqrt{5})$.

13. As in 12(a), using $c = 1$ avoids both 0 and $-\sqrt{p/q}$. 