Strategy (First Sylow Theorem)

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Let $G$ be a finite group and $p$ a prime such that $p^k || G$. Then $G$ has a subgroup of order $p^k$. 

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