# Recent Talks

"The Singular Value Decomposition and a Democratic Method of Orthogonalization"

The Fundamental Theorem of Arithmetic states that any positive integer $n$ can be factored uniquely as a product of primes. Factorization of real numbers (elements of $\mathbb{R}$) is hardly unique; if $x \in \mathbb{R}$ and $y \in \mathbb{R} \setminus \{0\}$, then $x$ can be factored as $x = by$ where $b = \frac{x}{y}$. If matrices $A$, $B \in \mathbb{R}^{n \times n}$ are non-singular, then $A$ can be factored as $A = BC$ where $C = B^{-1}A$. But what about non-singular matrices? Or even non-square matrices? There are several factorizations that can be applied to even singular matrices. Perhaps the most important of these is the singular value decomposition, or SVD.

We'll assume $m \geq n$. The (full) SVD of a $\mathbb{R}^{m \times n}$ matrix $A$ is $A = U\Sigma V^T$ where $U \in \mathbb{R}^{m \times m}$ and $V \in \mathbb{R}^{n \times n}$ are orthogonal, $V^T$ denotes the transpose of $V$, and $\Sigma$ is a diagonal matrix in $\mathbb{R}^{m \times n}$ possessing nonnegative entries, with the same rank as $A$. We will introduce the SVD from a geometric standpoint, using a $3 \times 3$ matrix as an example. We'll then eliminate some redundancy in the full SVD by introducing the *reduced* SVD, where $U \in \mathbb{R}^{m \times n}$ and $\Sigma \in \mathbb{R}^{n \times n}$. Note that $U$ is no longer an *orthogonal matrix*, but the columns of $U$ are orthogonal. Now $U$ and $V^*$ are of compatible size for the multiplication $UV^T$ to be defined. The matrix $T = UV^T \in \mathbb{R}^{m \times n}$ has orthogonal columns which are as close as possible to the columns of $A$. Besides the mathematical beauty of this method of orthogonalization of the column vectors of $A$, it possesses application in the sciences, which we'll discuss briefly.