MATH 141

Final Exam

May 9, 2005

NAME (please print legibly): ______________________________

Your University ID Number: ______________________________

- **No calculators are allowed on this exam.**
- **Please show all your work. You may use back pages if necessary. You may not receive full credit for a correct answer if there is no work shown.**
- **Answers such as $\frac{23\cdot 5}{30} - \frac{2^5}{3\cdot 34}$ are perfectly fine!! However you MUST simplify expressions such as $\sin(\pi/3)$.**

Part A		
QUESTION	VALUE	SCORE
1	30	
2	10	
3	15	
4	30	
5	15	
TOTAL	100	

Part B		
QUESTION	VALUE	SCORE
1	15	
2	15	
3	15	
4	15	
5	15	
6	10	
7	15	
TOTAL	100	

Part A

1. (30 pts) Limits. Calculate the following limits.

(a) $\displaystyle \lim_{x\to 5} \frac{x^2+25}{x+5}$

(b) $\displaystyle \lim_{h\to 0} \frac{\sqrt{4+h}-2}{h}$

(c) $\displaystyle \lim_{x\to 0} \frac{e^x - x - 1}{6x^2}$

(d) Find the following limits from the graph of $g(x)$ given below. Write DNE if the limit does not exist and is not $\pm\infty$.

$\displaystyle \lim_{x\to 2^-} g(x) =$____________ $\displaystyle \lim_{x\to 2^+} g(x) =$____________

$\displaystyle \lim_{x\to -2} g(x) =$____________ $\displaystyle \lim_{x\to 4} g(x) =$____________

2. (10 pts) Continuity. Find the constant k so that the function $g(x)$ is continuous at $x = 3$.

$$g(x) = \begin{cases} 3x^2 - 2x + k, & \text{if } x < 3 \\ \sin(\frac{\pi}{18}x), & \text{if } x \geq 3 \end{cases}$$

3. (15 pts) Definition of Derivative. Calculate the derivative of $f(x) = 3 - \frac{5}{6x}$ using the definition of the derivative function.

4. (30 pts) Calculating Derivatives Using Differentiation Formulas.

(a) Find $f'(x)$. $\qquad f(x) = \dfrac{3}{4\sqrt{x}} + \sin(4x) - e^{(x^3)}$

(b) Find $f'(x)$. $\qquad f(x) = \cos(5x)\ln(7x+2)$

(c) Find $\frac{dy}{dx}$. $\qquad (x^2 + y^2)^3 = x^3 y$

(d) Find y'. $\qquad y = \dfrac{3x^4 - 6x^2 + 2}{\sqrt[3]{x}}$

5. (15 pts) Application position/velocity. An object moves along a straight line and its position at time t is given by $s(t) = 2t^3 - 15t^2 + 24t$ where s is measured in feet and t in seconds.

(a) Find the average velocity of the object over the time interval $0 \leq t \leq 3$ seconds.

(b) Find the velocity of the object at time $t = 0$.

(c) At what time(s) is the object at rest?

(d) Find the total distance the object travels over the interval $0 \leq t \leq 10$.

Note: You do not need to algebraically simplify your answer.

Part B

1. (15 pts) Max/Min values of $f(x)$ on a Closed Interval.

Find the absolute maximum and absolute minimum values of

$$f(x) = x^3 - \frac{3}{2}x^2 - 6x$$

on the interval $[-2, 5]$.

The absolute minimum value is _________________________ which happens at ____________.

The absolute maximum value is _________________________ which happens at ____________.

2. (15 pts) Related Rates At noon, Ship A is 50 nautical miles due north of Ship B. Ship A is traveling west at 20 nautical miles per hour and Ship B is traveling east at 35 nautical miles per hour. How fast is the distance between the ships changing at 5PM?

3. (15 pts) Optimization. A farmer has 1000 feet of fencing to build two pens alongside her barn, one for sheep and one for pigs. Find the dimensions of the pens which maximize the area inside the pens for the animals.

4. (15 pts) Derivatives and the shape of the graph. Consider the function

$$f(x) = x^{2/3}(x-5)$$

.

Find the critical numbers of f.

Determine the intervals over which $f(x)$ is increasing.

Determine the intervals over which $f(x)$ is decreasing.

Determine the intervals over which $f(x)$ is concave up.

Determine the intervals over which $f(x)$ is concave down.

Graph the function $f(x)$ below and **label all local maxima, local minima and inflection points**.

5. (15 pts) Biology Application. Graph matching. Match the following scenarios to the graph pictured below which most closely represents the graph of the described population. Write the letter of the matching graph in the space provided to the left of each scenario.

_____________ A population of sea monkeys is increasing with an increasing growth rate.

_____________ A population of wolves is growing with a decreasing growth rate.

_____________ A population of hedgehogs has a constant positive growth rate.

_____________ A population of killer whales is decreasing with an increasing growth rate.

6. (10 pts) Calculating an Integral from the Definition. Note: You can check your answer to this problem at the end by using the Fundamental Theorem of Calculus (FTC), however you will earn zero points on this problem if your solution uses only the FTC.

$$\int_0^4 3 - x^2 \, dx = \lim_{n\to\infty} \left[\sum_{i=1}^{n} f(a_i)\Delta x \right]$$

Begin by drawing the graph of $f(x) = 3 - x^2$.

Fill in the blanks.

$\Delta x =$ ________________________ $a_i =$ ________________________

$f(a_i) =$ ________________________

Next simplify the following sum to a formula which is a function of n which does NOT contain summation notation. [You may wish to use one or more of the summation formulae given on the front page of this exam.]

$\sum_{i=1}^{n} f(a_i)\Delta x =$

Next, using the formula you found above, calculate the limit,

$\lim_{n\to\infty} \left[\sum_{i=1}^{n} f(a_i)\Delta x \right] =$

Hence, $\int_0^4 3 - x^2 \, dx =$

7. (15 pts) Calculating Integrals.

Calculate the following integrals. You do NOT have to algebraically simplify your solution.

(a) $\int_{2}^{10} 3x^4 - 2\sqrt{x} + 5 \; dx$

(b) $\int_{1}^{3} \frac{x^3 - 4x^{1/3}}{x^2} + 2\cos(x) \; dx$

(c) $\int_{1}^{5} f(x) \; dx$, where the graph of $f(x)$ is given below.