MATH 161Q

Midterm 2

November 5, 2002

NAME (please print legibly): _____

Your University ID Number: _____

- **No calculators are allowed on this exam.**
- **Please show all your work. You may use back pages if necessary. You may not receive full credit for a correct answer if there is no work shown.**
- To save time, you do NOT have to simplify any arithmetic. Answers such as $\frac{6}{7}3^5 - \frac{11}{120}3^2$ are perfectly fine.

QUESTION	VALUE	SCORE
1	12	
2	5	
3	12	
4	12	
5	10	
6	10	
7	24	
8	15	
TOTAL	100	

1. (12 pts) Use the **definition of the derivative function** to prove the following rule for differentiation.

For differentiable functions f and g: $\frac{d}{dx}(f(x)+g(x)) = \frac{d}{dx}(f(x)) + \frac{d}{dx}(g(x))$

2. (5 pts) TRUE or FALSE If the statement is true explain why it is true,

if the statement is false give an example why it is false.

If $\lim_{x\to a} f(x) = f(a)$, then it can be shown that the function f is differentiable at $x = a$.

3. (12 pts) Suppose an object moves along the y-axis you observe that its displacement, velocity, and acceleration satisfy the conditions:

(i) the initial displacement of the object at time equal to zero is y=0,
(ii) the initial velocity of the object is -1 meters/sec,
(iii) the object turns around at $t = 2$, and
(iv) the object has zero acceleration when $t = 4$.

Find the numbers a, b, c, and d so that the function $y = s(t) = at^3 + bt^2 + ct + d$ models the displacement of the object at time t.

For what time intervals is the object moving in the positive direction?

Find the total distance traveled by the object over the time interval $0 \leq t \leq 10$.

4. (12 pts)

The following equation describes a curve in the xy-plane.

$$x^2 + 9y^2 = 4xy + 7$$

Find all points on this curve (x_0, y_0), whose tangent line is horizontal.

5. (10 pts) a) Find the linear approximation of the function $f(x) = \sqrt[3]{x}$ at $x = 8$. Where is this approximation expected to be reasonable?

b) Use part a) to approximate the number $\sqrt[3]{8.2}$. Please simplify this answer.

6. (10 pts) Prove that $\dfrac{d}{dx}(\tan^{-1}(x)) = \dfrac{1}{1+x^2}$.

7. (24 pts) For each of the functions below find $\frac{dy}{dx}$.

a) $y = e^{3x^2+2x}$

b) $y = 2^x \cdot x^2$

c) $y = \ln(x^2 + \sin(x^3))$

d) $y = \dfrac{6x-1}{3-x^2}$

e) $y = \tan^{-1}(x^2 - 3)$

f) $y = x^{\sin(x)}$

8. (15 pts)

A trough is 100 ft long and its ends have the shape of isosceles triangles that are 30 ft across at the top and have a height of 10 ft. If the trough is filled with water at the rate of 90 ft^3/min, how fast is the water level rising when the water is 8 feet deep?