MATH 142 Midterm Exam #1

October 10, 2008

NAME:_____

- **No calculators are allowed on this exam.**
- **Answers such as $\frac{23\cdot 5}{30} - \frac{2^5}{3\cdot 34}$ are perfectly fine!! However you MUST simplify expressions such as $\sin(\pi/3)$.**
- **Please show all your work. You may use back pages if necessary. You may not receive full credit for a correct answer if there is no work shown.**
- **Please include all information about u-substitutions, and use correct mathematical grammar in the presentation of your solution.**

Problem	Points	Score
1	20	
2	30	
3	15	
4	15	
5	20	
total	100	

$$\sum_{i=1}^{n} a = a \cdot n \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

1. Definition of the Integral. Recall the definition of the definite integral for a continuous function $f(x)$ on the interval $[a, b]$ using right hand endpoints in the Riemann sum.

$$\int_a^b f(x)\ dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

Calculate the integral below using the definition of the integral and right hand endpoints in the Riemann sum.

$$\int_1^3 5 - x\ dx$$

(a) First, find the following quantities:

$\Delta x =$ _____ $x_i =$ _____ $f(x_i) =$ _____

(b) Next, using the quantities above and the summation formulas on the front page of the exam, simplify $\sum_{i=1}^{n} f(x_i)\Delta x$ into an expression without the summation notation.

(c) Last, evaluate the limit, $\lim_{n\to\infty}(\sum_{i=1}^{n} f(x_i)\Delta x)$.

Note: you can *check* your answer by using the Fundamental Theorem of Calculus.

2. Integrals. Evaluate the following definite and indefinite integrals.

(a) $\int (t^2 - \frac{7}{\sqrt{t}} + 45t)t^{-2}\ dt$

(b) $\int_{\sqrt{\frac{\pi}{12}}}^{\sqrt{\frac{\pi}{2}}} x \sin(3x^2)\ dx$

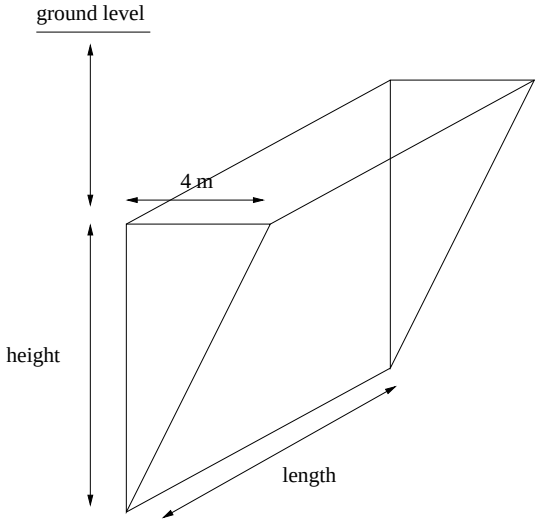
3. Let $f(x) = \int_0^x \frac{5-t}{t^4+7} dt$. Determine if the following statements are TRUE or FALSE. Justify your answers; unsupported answers will receive no credit.

(a) $f(0) = \frac{5}{7}$

(b) $f(5) < 0$

(c) f has a local maximum at $x = 5$

4. A gas station stores its fuel in a underground tank. The shape of the tank is pictured below. The height of the tank is 5 $meters$, length is 10 $meters$, and the top of the tank is located 8 $meters$ below ground. Using that the density of the fuel is 673 kg/m^3 and the acceleration due to gravity is 9.8 m/s^2, set up but DO NOT EVALUATE an integral that calculates the work required to empty the tank full of fuel.

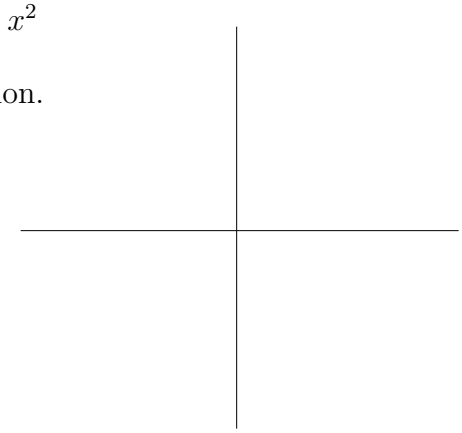


5. Volume. Note: On this problem, you can earn partial credit for parts (b)-(d) by sketching an arbitrary slice of the volume in the space provided in the left margin.

Consider the region, R which is bounded between the curves

$$y = 2x + 3 \qquad y = x^2$$

(a) Sketch the region and label the points of intersection.



(b) Write an integral for the volume of the solid formed by rotating the region R about the x-axis. DO NOT evaluate the integral.

(c) Write an integral for the volume of the solid formed by rotating the region R about the line $x = -3$. DO NOT evaluate the integral.

(d) Now consider the solid whose base is the region R and whose cross-sections above the xy-plane and perpendicular to the x-axis (i.e. slices parallel to the y-axis) are squares. Write an integral for the volume of this solid, but DO NOT evaluate the integral.