MATH 142

Final Exam

December 20, 2008

NAME (please print legibly): _____

Your University ID Number: _____

- **The first part of the final can replace your lowest midterm score, but it will also count towards your score on the final. If you skip it, you will get at most 100 points out of 200 on the final exam.**
- **No calculators are needed or allowed on this exam.**
- **Please put your final answers in the spaces provided.**
- **When integrating, put down all information you are using, such as substitutions or integration by parts.**
- **You do not need to simplify expressions such as $\frac{3}{4^2} + 26(3)^4 - \frac{\pi}{2}$, but you do need to evaluate expressions such as $\sin(\pi/4)$.**

Part A		
QUESTION	VALUE	SCORE
1	20	
2	10	
3	15	
4	30	
5	10	
6	15	
TOTAL	100	

Part B		
QUESTION	VALUE	SCORE
7	10	
8	10	
9	10	
10	20	
11	10	
12	10	
13	20	
14	10	
TOTAL	100	

Part A
Formulas:

$$\begin{aligned} \sin^2 x &= \frac{1}{2}\big(1 - \cos(2x)\big) \\ \cos^2 x &= \frac{1}{2}\big(1 + \cos(2x)\big) \end{aligned}$$

Expression	Substitution	
$\sqrt{a^2 - x^2}$	$x = a\sin\theta,$	$-\pi/2 \le \theta \le \pi/2$
$\sqrt{a^2 + x^2}$	$x = a\tan\theta,$	$-\pi/2 < \theta < \pi/2$
$\sqrt{x^2 - a^2}$	$x = a\sec\theta,$	$0 \le \theta < \pi/2$ or $\pi \le \theta < 3\pi/2$

$$\int_\alpha^\beta \frac{1}{2} r^2 \, d\theta$$

$$\int_a^b \sqrt{1 + (f'(x))^2} \, dx$$

$$\int \sec(x) \, dx = \ln|\sec(x) + \tan(x)| + C$$

1. (20 pts) (a) A graph the function $g(t)$ is given below. Let $F(x) = \int_0^x g(t)\ dt$.

(b) Is $F(1)$ positive or negative? Give reasons for your answer.

(c) Is $F(2)$ larger or smaller than $F(1)$? Give reasons for your answer.

(d) Is $F(2.25)$ larger or smaller than $F(2)$? Give reasons for your answer.

(e) At what value of x does the minimum value of $F(x)$ occur? Give reasons for your answer.

2. (10 pts) A solid of revolution has a volume, V, of 4 cubic units and a cross-sections of area $A(x)$, where x ranges from 0 to 1. The graph of $A(x)$ is given below, along with the graphs of three other functions. Which one is the graph of $A(x)$? Justify your answer.

3. (15 pts) The equation of a circle of radius r centered at the origin is $x^2 + y^2 = r^2$.

(a) Set up an integral that calculates the area inside one quarter of a circle of radius r.

(b) Evaluate the integral to prove that the area of a circle of radius r is given by $A = \pi r^2$.

4. (30 pts) Evaluate TWO of the following THREE integrals. If the integral is divergent, show why it is divergent. **Do NOT evaluate all three integrals. Clearly mark the integral you do NOT want me to grade.**

(a) $\displaystyle\int_0^{\pi/4} \tan(x)\sec^2(x)\,dx$

ANSWER: _____

(b) $\displaystyle\int_1^4 \frac{\ln(x)}{\sqrt{x}}\,dx$

ANSWER: _____

(c) $\displaystyle\int \frac{3x+2}{(x+2)(x+1)(x-1)}\,dx$

ANSWER: _____

5. (10 pts) Set up, but DO NOT EVALUATE, an integral which calculates the work required to empty the water in the tank pictured below through the pictured spigot. Recall that the density of water is $1000kg/m^3$ and the acceleration due to gravity is $9.8m/s^2$.

ANSWER: _____

6. (15 pts)

Give a careful sketch of the parametric curves given below on the axes provided. Label the point on each curve where $t = 0$ and put arrows on the curve indicating the direction of travel along the curve as t increases.

$$\begin{cases} x &= 3\cos(t) \\ y &= 3\sin(t) \end{cases}$$

$$\begin{cases} x &= t^2 \\ y &= 1 - t^2 \end{cases}$$

For each curve, label the point on the curve where $t = \pi/6$ and find the slope of the line tangent to the curve at $t = \pi/6$.

Part B

7. (10 pts) Determine whether each of the sequences given below is convergent or divergent. If it converges, find the limit.

(a) $a_n = \frac{\cos(n)}{n}$ for $n \geq 1$.

(b) { 3, 3.1, 3.14, 3.141, 3.1415, 3.14159, . . . }

8. (10 pts) Find $\sum_{n=1}^{\infty} a_n$ where a_n is the area of the triangle A_n in the diagram.

9. (10 pts) For each of the series given below determine whether or not it is **convergent or divergent.** If it is convergent, calculate the **sum** of the series.

(a) $15 + 10 + \frac{20}{3} + \frac{40}{9} + \frac{80}{27} + \dots$

ANSWER: _____

10. (20 pts) Determine whether the series converges or diverges. State the convergence test you are using, check the hypotheses of the test, and clearly state your conclusion.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{3}{\sqrt[3]{n}}$

ANSWER: _____

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{2-3n}{9+n}$

ANSWER: _____

11. (10 pts) If the following series is approximated by the sum of the first 10 terms give a bound on the error. If the series is divergent, state that the error is infinite.

$$\sum_{n=1}^{\infty} \frac{(-1)^n 7}{\sqrt{n+3}}$$

ANSWER: _____

12. (10 pts) Find the interval of convergence for the following power series.

$$\sum_{n=1}^{\infty} 5^n x^n$$

ANSWER: _____

13. (20 pts) (a) Find the Maclaurin series for $f(x) = e^x$ centered at $a = 0$.

(b) Find a power series representation for the function $f(x) = e^{-x^2}$

(b) Give an approximation of the value of the integral below with error less than 2^{-11}. Explain why the error is less than 2^{-11}

$$\int_0^{1/2} e^{-x^2}\, dx$$

ANSWER: _____

14. (10 pts) (a) Find the 3^{rd} degree Taylor polynomial, $T_3(x)$, for $f(x) = \sqrt{x}$ centered at $a = 4$. You do **NOT** need to simplify your answer by multiplying out the fractions.

ANSWER: _____

(b) Use the third degree Taylor polynomial to approximate the value of $\sqrt{4.2}$.

ANSWER: _____