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Math Symbols

$\wedge$ Conjunction	$\vee$ Disjunction	$\sim$ Negation
$\rightarrow$ Conditional	$\leftrightarrow$ Biconditional	
$\Rightarrow$ Implication	$\Leftrightarrow$ Equivalence	
$\forall$ For all	$\exists$ There exists	$\in$ in
$\therefore$ Therefore	■ QED	

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Rules of Valid Argumentation Involving Implication

Let  $P, Q, R,$  and  $S$  be statements,

i	$(P \rightarrow Q) \wedge P \Rightarrow Q$	Modus Ponens (Mode that affirms), used in Direct Proof
ii	$(P \rightarrow Q) \wedge \sim Q \Rightarrow \sim P$	Modus Tollens (Mode that denies), used in Proof by Contrapositive
iii	$(P \wedge Q) \Rightarrow P$	Specialization
iv	$(P \wedge Q) \Rightarrow Q$	Specialization
v	$P \Rightarrow P \vee Q$	Addition
vii	$(P \vee Q) \wedge \sim P \Rightarrow Q$	Modus Tollendo Ponens (also called Disjunctive Syllogism. Mode which, by taking away, affirms)
ix	$P \leftrightarrow Q \Rightarrow P \rightarrow Q$	Biconditional-Conditional
x	$P \leftrightarrow Q \Rightarrow Q \rightarrow P$	Biconditional-Conditional
xi	$(P \rightarrow Q) \wedge (Q \rightarrow P) \Rightarrow P \leftrightarrow Q$	Conditional-Biconditional
xii	$(P \rightarrow Q) \wedge (Q \rightarrow R) \Rightarrow P \rightarrow R$	Hypothetical Syllogism
xiii	$(P \rightarrow Q) \wedge (R \rightarrow S) \wedge (P \vee R) \Rightarrow (Q \vee S)$	Constructive Dilema

Rules of Valid Argumentation Involving Equivalence

Let  $P, Q,$  and  $R$  be statements,

i	$\sim(\sim P) \Leftrightarrow P$	Double negation, used in Proof by Contradiction
ii	$P \vee Q \Leftrightarrow Q \vee P$	Commutative Law
iii	$P \wedge Q \Leftrightarrow Q \wedge P$	Commutative Law
iv	$(P \vee Q) \vee R \Leftrightarrow P \vee (Q \vee R)$	Associative Law
v	$(P \wedge Q) \wedge R \Leftrightarrow P \wedge (Q \wedge R)$	Associative Law
vi	$P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$	Distributive Law
vii	$P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$	Distributive Law
viii	$P \rightarrow Q \Leftrightarrow \sim P \vee Q$	
ix	$P \rightarrow Q \Leftrightarrow \sim Q \rightarrow \sim P$	Contrapositive, used in proofs by Contrapositive
x	$P \leftrightarrow Q \Leftrightarrow Q \leftrightarrow P$	
xi	$P \leftrightarrow Q \Leftrightarrow (P \rightarrow Q) \wedge (Q \rightarrow P)$	
xii	$\sim(P \wedge Q) \Leftrightarrow \sim P \vee \sim Q$	De Morgan's Law
xiii	$\sim(P \vee Q) \Leftrightarrow \sim P \wedge \sim Q$	De Morgan's Law
xiv	$\sim(P \rightarrow Q) \Leftrightarrow P \wedge \sim Q$	
xv	$\sim(P \leftrightarrow Q) \Leftrightarrow (P \wedge \sim Q) \vee (\sim P \wedge Q)$	

Rules of Valid Argumentation Involving Quantifiers

Let  $P(x)$  be a predicate,

$[(\forall x \in U)P(x)] \Rightarrow P(a)$ where $a$ is arbitrary, we can chose it to be whatever we want	Universal Instantiation
$[(\exists x \in U)P(x)] \Rightarrow P(b)$ where $b$ is some particular element of $U,$ $b$ can not have appeared before	Existential Instantiation
$P(c)$ where $c$ is an arbitrary element of $U \Rightarrow [(\forall x \in U)P(x)]$	Universal Generalization
$P(d)$ where $d$ is some particular element of $U \Rightarrow [(\exists x \in U)P(x)]$	Existential Generalization

Thanks to Professor McNicholas for this handy one page summary of the Laws of Inference and Equivalence.