Laws of Inference

Modus Ponens:

$$\begin{array}{ll} & p \to q \\ & p \\ \therefore & q \end{array}$$

Modus Tollens:

$$\begin{array}{ll} & p \to q \\ & \sim q \\ \therefore & \sim p \end{array}$$

Specialization:

$$\begin{array}{ll} & p \wedge q \\ \therefore & p \end{array}$$

Generalization:

$$\begin{array}{ll} & p \\ \therefore & p \vee q \end{array}$$

Conjunction:

$$\begin{array}{ll} & p \\ & q \\ \therefore & p \wedge q \end{array}$$

Elimination:

$$\begin{array}{ll} & p \vee q \\ & \sim q \\ \therefore & p \end{array}$$

Transitivity:

$$\begin{array}{ll} & p \to q \\ & q \to r \\ \therefore & p \to r \end{array}$$

Division into cases:

$$\begin{array}{ll} & p \vee q \\ & p \to r \\ & q \to r \\ \therefore & r \end{array}$$

Contradiction rule:

$$\begin{array}{ll} & \sim p \to c \\ \therefore & p \end{array}$$

Constructive Dilemma:

$$\begin{array}{ll} & p \to q \\ & r \to s \\ & p \vee r \\ \therefore & q \vee s \end{array}$$

Biconditional-Conditional:

$$\begin{array}{ll} & p \leftrightarrow q \\ \therefore & p \to q \end{array}$$

Conditional-Biconditional:

$$\begin{array}{ll} & p \to q \\ & q \to p \\ \therefore & p \leftrightarrow q \end{array}$$

Laws of Inference Involving Quantifiers

Universal Instantiation:

$\forall\, x \in U,\ P(x)$

$\therefore P(a)$

where a is any member of U

Existential Instantiation:

$\exists\, x \in U,\ P(x)$

$\therefore P(b)$

where b is a particular member of U

Universal Generalization:

for an arbitrary element c of U

$P(c)$

$\therefore\ \forall\, x \in U,\ P(x)$

Existential Generalization:

for an element d of U

$P(d)$

$\therefore\ \exists\, x \in U,\ P(x)$

Universal Modus Ponens:

$\forall\, x \in U,\ P(x) \to Q(x)$

$P(a),$ for some particular a in U

$\therefore\ Q(a)$

Universal Modens Tollens:

$\forall\, x \in U,\ P(x) \to Q(x)$

$\sim Q(a),$ for some particular a in U

$\therefore \sim P(a)$

Universal Transitivity:

$$\begin{array}{ll} & \forall\, x \in U,\ P(x) \to Q(x) \\ & \forall\, x \in U,\ Q(x) \to R(x) \\ \therefore & \forall\, x \in U,\ P(x) \to R(x) \end{array}$$

Homework Problems:

Please refer back to the derivation example done in class. In general, a derivation is a step-by-step logical justification for an argument. At each step only one Law of Inference is used and the name of the law is cited along with indicating the previous lines in the argument (a premise or a previously justified statement) to which the law was applied.

Problem A. For each of the following arguments, if it is valid, use the Laws of Inference to give a derivation and if it is not valid, show why.

- If Fishville is boring, then it is hard to find. If Fishville is not small, then it is not hard to find. Fishville is boring. Therefore Fishville is small.
- If Susan likes fish, then she likes onions. If Susan does not like garlic, then she does not like onions. If she likes garlic, then she likes guavas. She likes fish or she likes cilantro. She does not like guavas. Therefore, Susan likes cilantro.

Problem B. Use the Laws of Inference to write a derivation for each of the following arguments.

- Every baby that eats will make a mess a drool. Every baby that drools will smile. There is a baby who eats and screams. Therefore there is a baby who smiles.
- Every cockroach that is clever eats garbage. There is a cockroach that likes dirt and does not like dust. For each cockroach, it is not the case that it likes dirt or eats garbage. Therefore there is a cockroach such that it is not the case that if it is not clever then it likes dust.