

Prove the following lemma.

Lemma: Let $n, k \in \mathbb{N}$. Then $\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}$.

Use mathematical induction and the lemma above to prove the Binomial Theorem. Hint: In the inductive step collect coefficients of $a^{k+1-i}b^i$.

Binomial Theorem Let $a, b \in \mathbb{R}$ and $n \in \mathbb{N}$. Then

$$(a + b)^n = \sum_{i=0}^n \binom{n}{i} a^{n-i} b^i$$