Suppose V is a subspace of $\mathbb{R}^n$, and V has more than the zero vector. A ***basis*** of V is a set of vectors that are both linearly independent and span V.

Find a basis for these subspaces:

1. The line $y = 3x$ in $\mathbb{R}^2$.
2. The plane $y = 3x$ in $\mathbb{R}^3$.
3. The plane $z = 3x + 4y$ in $\mathbb{R}^3$.
4. $\mathbb{R}^3$.

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Theorems about bases

1. A set of vectors $\{\vec{v}_1, \ldots, \vec{v}_k\}$ is a basis for V if and only if every vector in V can be written as a linear combination of the vectors $\{\vec{v}_1, \ldots, \vec{v}_k\}$ in exactly one way.
2. Every subspace V with more than the zero vector has a basis.
3. Every basis of V has the same number of vectors.

The ***dimension*** of V, $\dim(V)$, is the number of vectors in a basis of V.

Properties of basis and dimension

Suppose V is a subspace of $\mathbb{R}^n$ and $S = \{\vec{v}_1, \ldots, \vec{v}_k\}$ is a set of k vectors in V.

1. If S is linearly independent then S is a subset of a basis of V.
2. If S spans V then a subset of S is a basis of V.
3. If S is linearly independent then $k \leq \dim(V)$.
4. If S spans V then $\dim(V) \leq k$.
5. If $\dim(V) < k$ then S is linearly dependent.
6. If $k < \dim(V)$ then S doesn't span V.
7. If S is linearly independent and $k = \dim(V)$ then S is a basis of V.
8. If S spans V and $k = \dim(V)$ then S is a basis of V.
9. If V is contained in another subspace W then $\dim(V) \leq \dim(W)$.
10. If V is contained in another subspace W and $\dim(V) = \dim(W)$ then $V = W$.