Questions

Suppose $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$.

1. If $E_1 A = \begin{bmatrix} d & e & f \\ a & b & c \\ g & h & i \end{bmatrix}$, what is E_1?
2. If $E_2 A = \begin{bmatrix} 3a & 3b & 3c \\ d & e & f \\ g & h & i \end{bmatrix}$, what is E_2?
3. If $E_3 A = \begin{bmatrix} a & b & c \\ d+3a & e+3b & f+3c \\ g & h & i \end{bmatrix}$, what is E_3?

Properties of Elementary Matrices

1. If B is obtained from an $m \times n$ matrix A by an elementary row operation, then $B = EA$, where E is the matrix obtained from I_m by the same elementary row operation.
2. If E is an $n \times n$ elementary matrix, then E is invertible, and E^{-1} is the elementary matrix that transforms E back into I_n.
3. An $n \times n$ matrix A is invertible if and only A is the product of elementary matrices.

Amazing Awesome Unifying Invertible Matrix Theorem

Theorem

Suppose A is an $n \times n$ matrix. The following are equivalent.

1. *A is invertible.*
2. *A is the product of elementary matrices.*
3. *The reduced row echelon form of A is I_n.*
4. $\operatorname{rank}(A) = n$.
5. *$A\vec{x} = \vec{0}$ has only the solution $\vec{x} = \vec{0}$.*
6. *$A\vec{x} = \vec{b}$ is consistent for all $\vec{b} \in \mathbb{R}^n$.*
7. *$A\vec{x} = \vec{b}$ has exactly one solution for all $\vec{b} \in \mathbb{R}^n$.*
8. *There is an $n \times n$ matrix C such that $CA = I_n$.*
9. *There is an $n \times n$ matrix D such that $AD = I_n$.*
10. *A^T is invertible.*