Four different ways of computing the matrix product *AB*:

- The entry in the *i*th row and *j*th column of *AB*, denoted (*AB*)_{*ij*}, is the dot product of the *i*th row of *A* with the *j*th column of *B*.
- 2 $A\vec{x} = x_1\vec{a}_1 + x_2\vec{a}_2 + \cdots + x_n\vec{a}_n$, where the vectors $\vec{a}_1, \ldots, \vec{a}_n$ are the columns of A.
- 3 $AB = \begin{bmatrix} A\vec{b}_1 A\vec{b}_2 \cdots A\vec{b}_n \end{bmatrix}$, where the vectors $\vec{b}_1, \ldots, \vec{b}_n$ are the columns of B.

 $AB = \begin{vmatrix} r_1 B \\ \vec{r}_2 B \\ \vdots \\ \vec{r}_{r-B} \end{vmatrix}$ where the vectors $\vec{r}_1, \ldots, \vec{r}_n$ are the rows of *A*.

- To multiply AB, we need the number of columns of A equal to the number of rows of B.
- 2 Most of the time $AB \neq BA$. (Matrix multiplication is not commutative.)
- Sut (AB)C = A(BC) as long as you keep them in the same order. (Matrix multiplication is associative.)

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \cdot \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

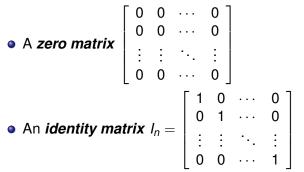
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- $A\vec{x}$ works as a matrix product if \vec{x} is a column vector.
- Solution Multiplying by a scalar is commutative: $A(c\vec{x}) = c(A\vec{x}) = (cA)\vec{x}$.
- $A(\vec{x} + \vec{y}) = A\vec{x} + A\vec{y}$ and A(B + C) = AB + AC. (Matrix multiplication is distributive over matrix addition.)

True or False?

- If AB and BA are both defined, then A and B are both square matrices.
- If B has a column of zeros, then so does AB.
- If A has a column of zeros, then so does AB.
- If A has two rows repeated, then so does AB.
- If A has two columns repeated, then so does AB.
- **(**) If AB = 0 then A = 0 or B = 0.
- If AC = BC then A = B.

Special matrices and other matrix operations:



- The *transpose* of *A*, A^{T} is formed by swapping rows and columns of *A*, equivalently, reflecting *A* across its main diagonal, equivalently, $(A^{T})_{ij} = (A)_{ji}$.
- The *trace* of *A*, tr(*A*) is the sum of the entries on the main diagonal of *A*.
- An $n \times n$ matrix A is *invertible* or *nonsingular* if there is a matrix A^{-1} (the *inverse* of A) such that $AA^{-1} = I_n$ and $A^{-1}A = I_n$.

$$(AB)^T = B^T A^T.$$

- 2 If A is invertible, then A^T is invertible, and $(A^T)^{-1} = (A^{-1})^T$.
- If A is an invertible matrix, then A^{-1} is invertible, and $(A^{-1})^{-1} = A$.
- If A and B are invertible $n \times n$ matrices, then AB is invertible, and

$$(AB)^{-1} = B^{-1}A^{-1}.$$

So If A is an invertible $n \times n$ matrix and k is a positive integer, then A^k is invertible, and

$$(A^k)^{-1} = (A^{-1})^k.$$

If A is an invertible matrix, then the equation $A\vec{x} = \vec{b}$ has the unique solution $\vec{x} = A^{-1}\vec{b}$.