1. Given a matrix $A$, the equation $A\mathbf{x} = \mathbf{b}$ might have 0 solutions for all $\mathbf{b}$, or 1 solution for all $\mathbf{b}$, or $\infty$ solutions for all $\mathbf{b}$, or 0 solutions for some choices of $\mathbf{b}$ and 1 solution for others, or 0 solutions for some choices of $\mathbf{b}$ and $\infty$ solutions for others, or 1 solution for some $\mathbf{b}$ and $\infty$ solutions for others, or 0, 1, or $\infty$ solutions for different choices of $\mathbf{b}$. (7 different combinations in all.)

Which of these combinations are actually possible? For those that are, give an example matrix and explain why it works. For those that aren’t, explain why not.